

## ❖ *Quantified Basic Sentences* ❖

### 5.6. A Modest Deployment of Quantifiers: Quantified Basic Sentences

**1. Quantified Basic Sentences.** All three of the quantified language we'll develop build on the language of sentence logic plus name atoms (a predicate letter followed by a name letter). So we start this chapter with the following construction rules in effect.

#### **Atomic Sentences:**

- A1. Sentence letters are atomic sentence
- A2. Name atoms (predicate letter followed by name letter) are atomic sentences.

#### **Formal Sentences:**

- 1. Atomic sentences are formal sentences.
- 2. If  $\bullet$  is a formal sentence, then  $\sim\bullet$  is a formal sentence.
- 3. If  $\bullet$  and  $\blacktriangle$  are formal sentences, then  $(\bullet \wedge \blacktriangle)$  is a formal sentence.
- 4. If  $\bullet$  and  $\blacktriangle$  are formal sentences, then  $(\bullet \vee \blacktriangle)$  is a formal sentence.
- 5. If  $\bullet$  and  $\blacktriangle$  are formal sentences, then  $(\bullet \rightarrow \blacktriangle)$  is a formal sentence.
- 6. If  $\bullet$  and  $\blacktriangle$  are formal sentences, then  $(\bullet \leftrightarrow \blacktriangle)$  is a formal sentence.

The first, and smallest, addition to these sentences yields the language of **Quantified Basic Sentences**. This lives up to its name by featuring **quantified basics**, strung together as sentences using the above molecular rules (2) through (6).

In Chapter Two we defined a **basic** as a sentence letter or its negation<sup>1</sup> – e.g., “P,” “~P,” “Q,” “~Q,” etc. To these we’ll add a new sort of atom, a **variable atom**: a predicate letter followed by a variable – e.g., “Gx,” “Hy,” etc. And then the new sort of **basics** under study here will be variable atoms and their negations – e.g., “Gx,” “~Gx,” “Hy,” “~Hy,” and so on. Call these **variable basics**.

We won’t add variable basics to the above list of formal sentences, because in this sub-language they’ll never show up on their own, only with a quantifier featuring the same variable.<sup>2</sup> The result will be the **quantified basics**.

### Variable Basic:

1. If ● is a predicate letter followed by a variable, then ● is a variable basic.
2. If ● is a predicate letter followed by a variable, then ~● is a variable basic.

### Quantified Basic:

1. If ★ is a variable and ● is a variable basic using ★, then  $\exists \star \bullet$  is a **quantified basic**.<sup>3</sup>
2. If ★ is a variable and ● is a variable basic using ★, then  $\forall \star \bullet$  is a **quantified basic**.

We can then add a clause to our construction rules counting quantified basics as formal sentences.

## 7. Quantified basics are formal sentences.

So all the following, being quantified basics, now count as formal sentences.

$\forall x Gx$	$\exists x Gx$	$\forall y Gy$	$\exists x Hx$
$\forall x \sim Gx$	$\exists x \sim Gx$	$\forall y \sim Gy$	$\exists x \sim Hx$

<sup>1</sup> In 2.26 §1.

<sup>2</sup> Moreover, even in later parts of this chapter (and throughout the next) we won’t count variable atoms or variable basics as **sentences** at all. See 5.11 for details.

<sup>3</sup> The ★ symbol is pronounced “star”. It’s used here as a generic blank which any **variable** can fill.

But then, by our construction rules **negations, conjunctions, disjunctions, conditionals, and biconditionals of these are also formal sentences**. So all the following now count as formal sentences.

$$\begin{array}{ll} \sim \forall x Gx & (Ga \rightarrow \exists x Gx) \\ (\exists x Gx \wedge \exists x \sim Gx) & ((\sim Ga \vee \sim Ha) \leftrightarrow \sim \forall x Gx) \\ (\forall x Gx \vee \sim \forall x \sim Gx) & (P \wedge \exists x \sim Gx) \end{array}$$

**Quantified Basic Sentences (QBSs)** get their name from being **sentences** built up from **quantified basics** (among other things).<sup>4</sup> The following construction rules set out the sentences of the **QBS Language**.

### Language of Quantified Basic Sentences (QBSs)

#### Atomic Sentences:

- A1. Sentence letters are atomic sentence
- A2. A predicate letter followed by a name letter is an atomic sentence.

#### Formal Sentences:

- 1. Atomic sentences are formal sentences.
- 2. If  $\bullet$  is a formal sentence, then  $\sim \bullet$  is a formal sentence.
- 3. If  $\bullet$  and  $\blacktriangle$  are formal sentences, then  $(\bullet \wedge \blacktriangle)$  is a formal sentence.
- 4. If  $\bullet$  and  $\blacktriangle$  are formal sentences, then  $(\bullet \vee \blacktriangle)$  is a formal sentence.
- 5. If  $\bullet$  and  $\blacktriangle$  are formal sentences, then  $(\bullet \rightarrow \blacktriangle)$  is a formal sentence.
- 6. If  $\bullet$  and  $\blacktriangle$  are formal sentences, then  $(\bullet \leftrightarrow \blacktriangle)$  is a formal sentence.
- 7. **Quantified basics are formal sentences.**<sup>5</sup>

<sup>4</sup> So don't misread that label: quantified basic sentences (QBSs) aren't **basic sentences** that get **quantified**. (Since variable basics don't count as sentences at all, they can't count as 'basic sentences'.) QBSs are **sentences** built up from **quantified basics** (among other things).

<sup>5</sup> Since quantified basics aren't here built up from smaller **sentences**, their construction rule might seem more suited for the list of **atomic** sentences. We place the rule here to better match the rules of later, larger (sub-)languages where quantifier construction rules are recursive, and definitely not atomic.

### Summary: Quantified Basic Sentences

- A **variable atom** is a predicate letter followed by a variable.
- A **variable basic** is a variable atom or negation of a variable atom.
- **Quantified Basic:**
  1. If  $\star$  is a variable and  $\bullet$  is a variable basic using  $\star$ , then  $\exists \star \bullet$  is a **quantified basic**.
  2. If  $\star$  is a variable and  $\bullet$  is a variable basic using  $\star$ , then  $\forall \star \bullet$  is a **quantified basic**.
- Since **quantified basics are formal sentences**, they can legally appear as building blocks in larger, molecular sentences (wherever a sentence letter can appear legally).
- The revised formal language, with quantified basics added, forms the **language of quantified basic sentences (QBS Language)**.